Linear Algebra and Applications
List 5
Deadline: June 3, 2020

The list below includes exercises from the textbook and some additional problems. Solving all problems is strongly recommended. The problems that must be written and sent to the monitor (scanned or photographed) by email **jose14manuel14@gmail.com by June 3** are: **Exercises: 7,12,20 (Chapter 7), 4,10,12 (Chapter 8) and Problems: 1,3,5.**

List of Problems

Textbook Exercises: 7,8,11,12,13,14,15,16,17,18,19,20,21 (Chapter 7) 1,4,8,10,11,12,13 (Chapter 8)

Problem 1. *Let V be the space of complex $n \times n$ matrices with the inner product $(A, B) = tr(AB^*)$. For each M in V, let T_M be the linear operator defined by $T_M(A) = MA$. Show that T_M is unitary if and only if M is a unitary matrix.*

Problem 2. *Let V be a finite-dimensional inner product space, and let W be a subspace of V. Then $V = W \oplus W^{\perp}$, that is, each α in V is uniquely expressible in the form $\alpha = \beta + \gamma$, with β in W and γ in $W^{\perp}$. Define a linear operator U by $U\alpha = \beta - \gamma$.*

- *a). Prove that U is both self-adjoint and unitary.*
- *b). If $V = \mathbb{R}^3$ with the standard inner product and W is the subspace spanned by $(1, 0, 1)$, find the matrix of U in the standard ordered basis.*

Problem 3. *Let V be a finite-dimensional inner product space. For each α, β in V, let $T_{\alpha,\beta}$ be the linear operator on V defined by $T_{\alpha,\beta}(\gamma) = (\gamma, \beta)\alpha$. Show that*

- *a). $T^*_{\alpha,\beta} = T_{\beta,\alpha}$.*
- *b). $T_{\alpha,\beta}T_{\gamma,\delta} = T_{\alpha,(\beta,\gamma)\delta}$.*
- *c). Under what conditions is $T_{\alpha,\beta}$ self-adjoint?*

Problem 4. *For*

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{pmatrix}$$

there is a real orthogonal matrix P such that $P^TAP = D$ is diagonal. Find P and D.

Problem 5. *Let A be a $n \times n$ matrix and denote the eigenvalues of the matrix $B = A^TA$ by λ_i $i = 1, 2, ..., n$. Then,*

$$\| A \|_2 = \max_{i=1}^{n} \lambda_i^{1/2},$$

where $\| . \|_2$ is the standard Euclidean norm in $\mathbb{R}^n$: $\| x \|_2 = \sqrt{x_1^2 + x_2^2 + ... + x_n^2}$.

Problem 6. *Let V be a finite-dimensional inner product space, and let E be an idempotent linear operator on V, i.e., $E^2 = E$. Prove that E is self-adjoint if and only if $EE^* = E^*E$.*

Problem 7. *If T is a unitary ans S is a self-adjoint operator, prove that TST^{-1} is a self-adjoint operator.*

Problem 8. *Let V be real inner product space consisting of the space of real-valued continuous functions on the interval $[-1, 1]$, with the inner product*

$$(f, g) = \int_{-1}^{1} f(t)g(t)dt.$$

Let W be the subspace of odd functions. Find the orthogonal complement of W, that is, $W^\perp$.