Problem Set 1

March 28, 2021

<u>Deadline</u>: April 1st

All problems from Lax's *Linear Algebra and its Applications*, as well as the problems from this set, are recommended.

These problems from Lax should be **sent by e-mail** (scanned, typed or photographed) to juliahdomingues@hotmail.com:

Chapter 1: 12, 19, 21

Chapter 2: 1, 2, 7

From this set: 3

Exercise 1

Let $V = \{(a, b) : a, b \in \mathbb{R}^+\}$. Define addition on V by

$$(a_1, b_1) + (a_2, b_2) = (a_1 a_2, b_1 b_2)$$

Further, define scalar multiplication for $c \in \mathbb{R}$ by

$$c \cdot (a, b) = (a^c, b^c)$$

Prove that V, with these operations, is a vector space over $\mathbb{R}$

Exercise 2

Let V be a vector space, $\mathcal{F}$ a collection of subspaces of V with the following property: If $X, Y \in \mathcal{F}$, then there exists a $Z \in \mathcal{F}$ such that $X \cup Y \subset Z$. Prove that $\bigcup_{U \in \mathcal{F}} U$ is a subspace of V.

Exercise 3

Let $V = \{a_0 + a_1 x + a_2 x^2 | a_i \in F\}$ and $f, g, h \in V'$ be defined by

$$f(p) = p(-1), \quad g(p) = p(0), \quad h(p) = p(1), \tag{3.1}$$

with $p(x) \in V$. Show that f, g, h is basis for V'.

Exercise 4

Let $X = C[a, b]$, ($a, b \in \mathbb{R}$ and $a < b$) be the vector space of all real-valued continuous functions over the interval $[a, b]$ and

$$Y = \{f \in X | \int_a^b f(t)dt = 0\}$$

a) Prove that if $\mathbb{R}$ is identified with the set of all constant functions over $[a, b]$ then $X = \mathbb{R} \oplus Y$.

b) For $a = 0, b = 1$, and $f(t) = t^2 + t - 1$, find the unique $c \in \mathbb{R}$ and $g \in Y$ such that $f(t) = c + g(t)$ for all $t \in [0, 1]$.