Problem Set 3

April 22, 2021

<u>Deadline</u>: May 6th

All problems from Lax's *Linear Algebra and its Applications*, as well as the problems from this set, are recommended.

These problems from Lax should be **sent by e-mail** (scanned, typed or photographed) to juliahdomingues@hotmail.com:

Chapter 5: Exercise 11

Chapter 6: Exercises 3, 10 and 11

From this set: Exercise 2

Exercise 1

Find the eigenvalues and eigenvectors of the matrix associated to the linear transformation T

a). $T(x, y) = (2x + 3y, 4x - 2y)$

b). $T(x, y, z) = (2x, 9x + 5y + 6z, 8x - 3y - z)$

Exercise 2

Let A be a $n \times n$ matrix satisfying the following diagonally dominant condition:

$$|a_{ii}| > \sum_{j \neq i} |a_{ij}|, \quad i = 1, \ldots, n$$

Show that $det(A) \neq 0$.

Exercise 3

Consider the matrix

$$A = \begin{pmatrix} a & b \\ b & c \end{pmatrix}$$

where $a, b, c \in \mathbb{R}$.

a) Prove that A has only real eigenvalues.

b) Under what conditions on a, b, c does A have a multiple eigenvalue?.

Exercise 4

Consider the matrix

$$\begin{pmatrix} 1 & 2 & 0 \\ 0 & 2 & 0 \\ -2 & -1 & -1 \end{pmatrix}$$

a). Use the characteristic polynomial of A and the Cayley-Hamilton theorem to find A^{-1}.

b). Use the characteristic polynomial of A and the Cayley-Hamilton theorem to find A^{10}.