Linear Algebra and Applications
List 3
Deadline: April 29, 2020

The list below includes exercises from the textbook and some additional problems. Solving all problems is strongly recommended. The problems that must be written and sent to the monitor (scanned or photographed) by email **jose14manuel14@gmail.com by April 29** are: **Exercises: 10,11,12 (Chapter 5), 3,8,10,11 (Chapter 6) and Problems: 1,9,10.**

List of Problems

Textbook Exercises: 8,9,10,11,12,13,16 (Chapter 5), 2,3,8,10,11,12,13 (Chapter 6).

Problem 1. *Consider the matrix*

$$\begin{pmatrix} 1 & 2 & 0 \\ 0 & 2 & 0 \\ -2 & -1 & -1 \end{pmatrix}$$

a). Use the characteristic polynomial of A and the Cayley-Hamilton theorem to find A^{-1}.

b). Use the characteristic polynomial of A and the Cayley-Hamilton theorem to find A^{10}.

Problem 2. *Find the eigenvalues and eigenvectors of the matrix associated to the linear transformation T*

a). $T(x,y) = (2x+3y, 4x-2y)$

b). $T(x,y,z) = (2x, 9x+5y+6z, 8x-3y-z)$

Problem 3. *Let A be an $n \times n$ matrix with characteristic polynomial*

$$p_A(s) = (-1)^n s^n + a_{n-1}s^{n-1} + ... + a_1 s + a_0$$

Then, A is invertible if and only if $a_0 \neq 0$.

Problem 4. *Let*

$$B = \begin{pmatrix} 8 & 12 & 0 \\ 0 & 8 & 12 \\ 0 & 0 & 8 \end{pmatrix}$$

Find a real matrix A such that $B = A^3$.

Problem 5. *Let A be a 2×2 matrix. Prove that $det(I + A) = 1 + det(A)$ if and only if $tr(A) = 0$.*

Problem 6. *Let A be a $n \times n$ matrix. Suppose that there exists some positive integer k so that $A^k = 0$. Prove that $A^n = 0$.*

Problem 7. *Let $q(s) = s^n + c_{n-1}s^{n-1} + ... + c_0$ be an arbitrary polynomial with coefficents in $\mathbb{R}$, and let*

$$A = \begin{pmatrix} 0 & 0 & 0 & \dots & -c_0 \\ 1 & 0 & 0 & \dots & -c_1 \\ 0 & 1 & 0 & \dots & -c_2 \\ \vdots & & \ddots & \ddots & \vdots \\ 0 & 0 & \dots & 1 & -c_{n-1} \end{pmatrix}$$

Prove that $p_A(s) = q(s)$. We call A the ***companion matrix*** *of $q(s)$. This shows that for any polynomial, there is a matrix whose characteristic polynomial is a constant multiple of the given polynomial.*

Problem 8. *Consider the matrix*

$$A = \begin{pmatrix} a & b \\ b & c \end{pmatrix}$$

where $a, b, c \in \mathbb{R}$.

- *a). Prove that A has only real eigenvalues.*
- *b). Under what conditions on a, b, c does A have a multiple eigenvalue?.*

Problem 9. *Suppose A and B are 3×3 matrices such that the first and second colums of A are same as the first and second columns of B. If $det(A) = 5$ and $det(B) = 2$, find $det(3A - 2B)$ and $det(3A + 2B)$.*

Problem 10. *Let A be a $n \times n$ matrix satisfying the following diagonally dominant condition:*

$$|a_{ii}| > \sum_{j \neq i} |a_{ij}|, \quad i = 1, ..., n$$

Show that $det(A) \neq 0$.