Linear Algebra and Applications
List 6
Deadline: June 17, 2020

The list below includes exercises from the textbook and some additional problems. Solving all problems is strongly recommended. The problems that must be written and sent to the monitor (scanned or photographed) by email **jose14manuel14@gmail.com by June 17** are: **Exercises: 2,3,7 (Chapter 9), 3,4,13 (Chapter 10) and Problems: 1,4,8.**

List of Problems

Textbook Exercises: 1,2,3,6,7 (Chapter 9) 3,4,5,7,12,13,14,15 (Chapter 10)

Problem 1. *Let A be a $n\times n$ real matrix. Suppose that A is nonnegative and consider the null set $S=\{x\in\mathbb{R}^n : x^tAx=0\}$. Prove that $x\in S$ if and only if $Ax=0$. What happens when A is indefinite such that there are nonzero vectors $x,y\in\mathbb{R}^n$ so that $x^tAx>0$ and $y^tAy<0$?.*

Problem 2. *Assume that A,B are $n\times n$ real matrices. Suppose that A and B are symmetric and the eigenvalues of A,B are greater than or equal to $a,b\in\mathbb{R}$, respectively. Show that the eigenvalues of $A+B$ are greater than or equal to $a+b$.*

Problem 3. *Consider the matrix*

$$A=\begin{pmatrix}1+i & 2 & -1\\ 2 & 1-i & 1\end{pmatrix}$$

a). Find the singular values of A.

b). Find a singular value decomposition of A.

Problem 4. *Let a and b be real numbers.*

a). If

$$K=\begin{pmatrix}0 & b\\ -b & 0\end{pmatrix}$$

Show that

$$e^{tK}=\begin{pmatrix}\cos(bt) & \sin(bt)\\ -\sin(bt) & \cos(bt)\end{pmatrix}$$

b). If

$$A=\begin{pmatrix}a & b\\ -b & a\end{pmatrix}$$

Show that

$$e^{tA}=\begin{pmatrix}e^{at}\cos(bt) & e^{at}\sin(bt)\\ -e^{at}\sin(bt) & e^{at}\cos(bt)\end{pmatrix}$$

Problem 5. *Suppose that A is a 2×2 matrix with repeated eigenvalue λ. Show that $e^{tA} = e^{\lambda t}(I + t(A - \lambda I))$.*

Problem 6. *Let*

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ -1 & -1 & 2 \end{pmatrix}$$

Find e^{tA}.

Problem 7. *Consider the quadratic form represent by the matrix*

$$A = \begin{pmatrix} 1 & a & 1/2 \\ a & 4 & 0 \\ 1/2 & 0 & 1 \end{pmatrix}$$

with $a \in \mathbb{R}$. Determine the value of a so that A is positive. Could it in any case be classified as non-negative?

Problem 8. *Let A, B be $n \times n$ real matrices. Suppose that A and B are positive. Prove that the eigenvalues of AB are all positive.*

Problem 9. *Let A, B be $n \times n$ real matrices. Suppose that A is positive and B is non-negative. Establish the inequality $det(A+B) \geq det(A)+det(B)$ and show that the equality holds only when $B = 0$.*