Arnold theory (1971)

theory of versal deformations of matrices.

$A \sim C^{-1}AC$ change of basis

Jordan normal form: $J = C^{-1}AC$

$$J = \begin{pmatrix} \Box & & & \\ & \Box & & \\ & & \ddots & \\ & & & \Box \end{pmatrix} \qquad \Box = \begin{pmatrix} \lambda & 1 & & \\ & \ddots & \ddots & \\ & & \ddots & 1 \\ & & & \lambda \end{pmatrix}$$

Jordan block

$A \longmapsto J$ reduction to normal form

$J(A)$ function. is NOT CONTINUOUS

$$A_0 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad A_1 = \begin{pmatrix} 0 & 0 \\ \varepsilon & 0 \end{pmatrix}$$

$$A(\varepsilon) = A_0 + \varepsilon A_1 = \begin{pmatrix} 0 & 1 \\ \varepsilon & 0 \end{pmatrix}.$$

$$A(0) \longrightarrow \begin{pmatrix} 0 & \boxed{1} \\ 0 & 0 \end{pmatrix}$$

$\varepsilon \neq 0 \qquad \lambda_{1,2} = \pm\sqrt{\varepsilon}$

discontinuum at $\varepsilon = 0$.

$$A(\varepsilon \neq 0) \longrightarrow \begin{pmatrix} \sqrt{\varepsilon} & \boxed{0} \\ 0 & -\sqrt{\varepsilon} \end{pmatrix}$$

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instead of J consider such different, such that it is regular (differentiable) w.r.t. A:

Consider a matrix $A(p)$ depending on parameter $p \in \mathbb{R}^n$ (smoothly).

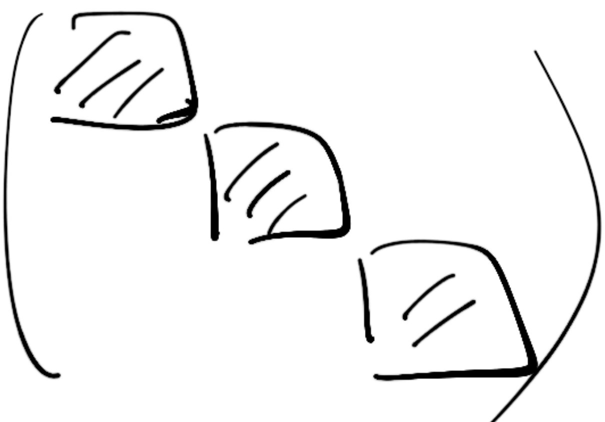
What is a simplest form of a matrix

$$\tilde{A}(p) = C^{-1}(p)\, A(p)\, C(p)$$

s.t. $\tilde{A}(p)$ is smooth and $C(p)$ is smooth

We know that $\tilde{A}$ cannot be Jordan form. Arnold derived the form of $\tilde{A}$ (called versal deformation).

The form of $\tilde{A}$ is block-diagonal, depends on $A(p=0)$;



each block correspond to one eigenvalue of $A(p=0)$. In particular, it can has the form

double eig.

$$\begin{pmatrix} \lambda_0 + a & 1 \\ b & \lambda_0 + a \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} \lambda_0 + a & b \\ c & \lambda_0 + d \end{pmatrix}$$