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Stability analysis of a rotating disk with rotating and nonrotating asymmetries in translatory and rotational degrees of freedom

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ABSTRACT

In this paper we consider the combined effect of rotating and non-rotating asymmetries on a rotor under non-conservative loading, motivated by technical applications in which breaking of symmetries may be used to avoid self-excited vibrations. We consider a rotating disk with two translatory degrees of freedom describing the displacements of the center of the disk with respect to the axis of rotation and two rotational degrees of freedom describing the tilting angles with respect to the axis of rotation. It is shown that breaking of symmetries of the rotating and non-rotating restoring terms has opposite effects for the stability behavior of translatory and rotational motions. Thus, if neither the translatory nor rotational degrees of freedom are dominant, it will be difficult to find robust designs against self-excited vibrations using the approach of breaking symmetries.

1. Introduction

In this paper we will study the combined effect of stationary and rotating asymmetries on a discrete rotor with translatory and rotational degrees of freedom (dof) under nonconservative loading. The stability behavior of rotors in frictional contact is very interesting and has been studied extensively using minimal models, e.g. [1–5], many of them originating from brake squeal problems or models of clutches. Common to the models used in the above-mentioned papers is that the stability behavior is governed by linear equations of motion with constant coefficients. From an eigenvalue analysis it can be shown that the models feature instabilities of the trivial solution and therefore self-excited vibrations arise. A major challenge for engineers dealing with noise, vibration and harshness problems is to find robust solutions against these unwanted self-excited vibrations. Such instabilities also occur in the undercritical speed range of the rotor on which we focus in this paper. Therefore a major goal of engineers is to get a profound understanding of the stability boundaries of the systems and therefore the parameter ranges in which self-excited vibrations occur. Since parameters are normally difficult to measure exactly, qualitative results are extremely valuable.

An engineering approach to avoid self-excited vibrations is to break symmetries of the systems. Patents based on this idea have been developed at least since the 60s, the authors refer in particular to [6–9], which are related to the suppression of brake squeal by the introduction of various geometrical imperfections destroying the rotational symmetry, e.g. uneven

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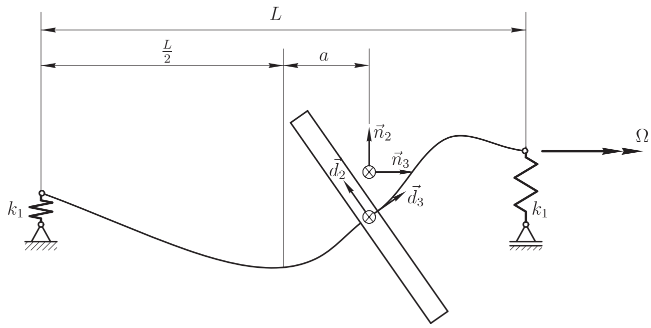

Fig. 1. Laval rotor with rotating and non-rotating stiffness.

chamfers or cooling channels. Technical papers aiming in this direction have been around at least since the 90s when Nishiwaki et al. designed brake disks [10] with the goal to prevent them from exhibiting standing wave squeal modes. In the models they attached point masses to the disks. Along similar lines Fieldhouse et al. in [11] designed asymmetric disk brakes by replacing the disk's cooling fins. Although the breaking of symmetries proved to be successful in experiments, the effect cannot be understood by analysis of models featuring constant coefficients.

Due to the fact that the equations of motion of asymmetric rotors in contact usually feature periodic coefficients, the amount and realization of asymmetry needed to avoid self-excited vibrations is difficult to quantify. In [12] the stabilizing effect of asymmetries has been shown by analytic perturbation theory along the lines of [13,14] allowing also to quantify the amount of asymmetry needed for the stabilization of a system. In [15] a rotor with two translatory dof has been analyzed and a stabilizing effect of an asymmetry in the bearings has been shown. For a rotor with two rotational dof it has been shown in [16] that a rotating asymmetry has a stabilizing effect.

In [17], the combined effect of static and rotating asymmetries has been studied for a rotor with two translatory dof in frictional contact. It has been shown that only the stationary asymmetry has a stabilizing effect in this case whereas the rotating asymmetry limits this effect. Based on this observation and in order to obtain a more complete picture, in this paper we study a system containing both translatory and rotational dof with stationary and rotating asymmetries. The analysis in this paper therefore covers the coupling of bending vibrations of a rotating shaft and tilting vibrations of a rotating disk in one model.

2. Physical model

In this section we introduce a parametric model containing translatory and rotational dof. Consider the Laval rotor shown in Fig. 1. The rotor consists of an elastic unround Euler Bernoulli beam (bending stiffnesses around the rotating axis: EI_1, EI_2) and a rigid disk of mass m and radius r attached at position $l=L/2+a$. The system has two translatory dof for the displacement of the center of the disk and two rotational dof allowing for a tilting. We assume that the disk has contacts such that nonconservative terms appear which can be dissipative or non-dissipative. We assume that the contact forces are small compared to the elastic restoring terms as is the case for brake squeal occurring at low brake pressures. Therefore the terms resulting from contact forces can be treated as perturbations. For the sake of brevity we just analyze the mathematical structure of the terms and will only briefly touch on their physical origination. Examples for possible settings can however be found in [16,17].

Before going into the formulation of the equations of motion we remark that if the disk is positioned in the middle of the beam at $l=\frac{1}{2}L$ ($a=0$), it is intuitively clear that translatory and rotational motion are decoupled.

3. Equations of motion for the system

At first we derive the general mathematical structure of the equations of motion in coordinate invariant form before the physical origination of the terms is presented. For the setup of the equations of motion there are various choices of generalized coordinates. In order to start from a coordinate invariant formulation we formulate the stationary and rotating restoring terms in terms of dyadics. The same we do for the inertia terms.

The general structure of the equations of motion is given by

$$\overrightarrow{\overrightarrow{M}}\,\ddot{\vec{p}}+(\overrightarrow{\overrightarrow{KC}}+\overrightarrow{\overrightarrow{N}})\vec{p}=\mathbf{0} \tag{1}$$

where the vector $\vec{p}$ consists of rotational components $\vec{\alpha}$ and translatory components $\vec{r}$, i.e.,

$$\vec{p}=\begin{pmatrix}\vec{r}\\ \vec{\alpha}\end{pmatrix}. \tag{2}$$

The inertia tensor $\overset{\rightrightarrows}{M}$ and the stiffness tensor $\overset{\rightrightarrows}{KC}$ relate forces/torques and displacements/displacement angles. The tensor $\overset{\rightrightarrows}{N}$ originates from non-conservative contact forces. The stiffness tensor is composed of a rotating component $\overset{\rightrightarrows}{C}$ and a stationary component $\overset{\rightrightarrows}{K}$.

As common springs, the restoring elements can in principle couple in two different ways. If they are aligned in series, the displacement vectors are equal and for the restoring force $\vec{F}$ we have the relation

$$\vec{r} = \overset{\rightrightarrows}{KC}^{-1} \cdot \vec{F} = (\overset{\rightrightarrows}{K}^{-1} + \overset{\rightrightarrows}{C}^{-1}) \cdot \vec{F}, \tag{3}$$

if they act in parallel we have

$$\vec{F} = \overset{\rightrightarrows}{KC} \cdot \vec{r} = (\overset{\rightrightarrows}{K} + \overset{\rightrightarrows}{C}) \cdot \vec{r}. \tag{4}$$

The resulting stiffness tensor is denoted by $\overset{\rightrightarrows}{KC}$. Combinations of the two cases can be treated in a straightforward manner. Since in our case the static and rotating stiffness are aligned in series we obtain

$$\overset{\rightrightarrows}{KC} = (\overset{\rightrightarrows}{K}^{-1} + \overset{\rightrightarrows}{C}^{-1})^{-1}. \tag{5}$$

In order to get a more detailed picture of the equations, we separate translatory and rotatory components of the matrices. The equations for a rotor with contact forces (and without damping) read

$$\begin{bmatrix} \overset{\rightrightarrows}{M}_{TT} & \overset{\rightrightarrows}{0} \\ \overset{\rightrightarrows}{0} & \overset{\rightrightarrows}{M}_{DD} \end{bmatrix} \begin{pmatrix} \ddot{\vec{r}} \\ \ddot{\vec{\alpha}} \end{pmatrix} + \begin{bmatrix} \overset{\rightrightarrows}{KC}_{TT} + \overset{\rightrightarrows}{N}_{TT} & \overset{\rightrightarrows}{KC}_{TD} \\ \overset{\rightrightarrows}{KC}_{DT} & \overset{\rightrightarrows}{KC}_{DD} + \overset{\rightrightarrows}{N}_{DD} \end{bmatrix} \begin{pmatrix} \vec{r} \\ \vec{\alpha} \end{pmatrix} = \vec{0}, \tag{6}$$

where the tensors $\overset{\rightrightarrows}{KC}_{ij}$ depend on the stationary and rotating bedding of the body. The translatory components are indicated by T, the rotatory ones by D. As mentioned previously, the vector $\vec{r}$ specifies the translatory displacement of the rotor, the vector $\vec{\alpha}$ describes the tilt angle of the disk with respect to its undeformed position. Usually the bedding consists of stationary and rotating restoring stiffnesses which can be coupled in different ways. The rotating restoring tensors are given by

$$\overset{\rightrightarrows}{C}_{TT} = c_{TT11} \vec{d}_1 \otimes \vec{d}_1 + c_{TT22} \vec{d}_2 \otimes \vec{d}_2, \tag{7a}$$

$$\overset{\rightrightarrows}{C}_{TD} = c_{TD11} \vec{d}_1 \otimes \vec{d}_1 + c_{TD22} \vec{d}_2 \otimes \vec{d}_2, \tag{7b}$$

$$\overset{\rightrightarrows}{C}_{DT} = c_{TD11} \vec{d}_1 \otimes \vec{d}_1 + c_{TD22} \vec{d}_2 \otimes \vec{d}_2, \tag{7c}$$

$$\overset{\rightrightarrows}{C}_{DD} = c_{DD11} \vec{d}_1 \otimes \vec{d}_1 + c_{DD22} \vec{d}_2 \otimes \vec{d}_2, \tag{7d}$$

the stationary restoring tensors are given by

$$\overset{\rightrightarrows}{K}_{TT} = k_{TT11} \vec{n}_1 \otimes \vec{n}_1 + k_{TT22} \vec{n}_2 \otimes \vec{n}_2, \tag{8a}$$

$$\overset{\rightrightarrows}{K}_{TD} = k_{TD11} \vec{n}_1 \otimes \vec{n}_1 + k_{TD22} \vec{n}_2 \otimes \vec{n}_2, \tag{8b}$$

$$\overset{\rightrightarrows}{K}_{DT} = k_{TD11} \vec{n}_1 \otimes \vec{n}_1 + k_{TD22} \vec{n}_2 \otimes \vec{n}_2, \tag{8c}$$

$$\overset{\rightrightarrows}{K}_{DD} = k_{DD11} \vec{n}_1 \otimes \vec{n}_1 + k_{DD22} \vec{n}_2 \otimes \vec{n}_2, \tag{8d}$$

where the vectors $\vec{d}_1$, $\vec{d}_2$ and $\vec{n}_1$, $\vec{n}_2$ are shown in Fig. 1.

For the nonconservative forces we assume

$$\overset{\rightrightarrows}{N}_{TT} = n_T \vec{n}_1 \otimes \vec{n}_2 - n_T \vec{n}_2 \otimes \vec{n}_1, \tag{9a}$$

$$\overset{\rightrightarrows}{N}_{DD} = n_D \vec{n}_1 \otimes \vec{n}_2 - n_D \vec{n}_2 \otimes \vec{n}_1. \tag{9b}$$

Note that due to the skew symmetry $\overset{\rightrightarrows}{N}_{TT}$ and $\overset{\rightrightarrows}{N}_{DD}$ are invariants with respect to rotation. This means that the terms can be assumed to be constant without loss of generality because the contacts are in fact constant either in a stationary or a rotating frame. At this point we do not specify damping terms since they can have various forms. In the numerical calculations we will however use a small damping matrix proportional to the stiffness matrix.

The coefficients of the dyadics for the rotating stiffness can be found from the tabulated expressions for the elastic line and are given as

$$c_{TT11} = \frac{3\,EI_2\,L(L^2 - 3\,L\,l + 3\,l^2)}{l^3(L-l)^3}, \tag{10a}$$

$$c_{TT22} = \frac{3\,EI_1\,L(L^2 - 3\,L\,l + 3\,l^2)}{l^3(L-l)^3}, \tag{10b}$$

$$c_{TD11} = -\frac{3\,EI_2\,L(L-2\,l)}{l^2(L-l)^2}, \tag{10c}$$

$$c_{TD22} = -\frac{3\,EI_1\,L(L-2\,l)}{l^2(L-l)^2}, \tag{10d}$$

$$c_{DD11} = \frac{3\,EI_2\,L}{l(L-l)}, \tag{10e}$$

$$c_{DD22} = \frac{3\,EI_1\,L}{l(L-l)}. \tag{10f}$$

The coefficients for the static restoring terms can be calculated in a straightforward way and are given as

$$k_{TT11} = 2\,k_1, \tag{11a}$$

$$k_{TT22} = 2\,k_2, \tag{11b}$$

$$k_{TD11} = k_1(L-2\,l), \tag{11c}$$

$$k_{TD22} = k_2(L-2\,l), \tag{11d}$$

$$k_{DD11} = k_1\,l^2 + k_1(L-l)^2, \tag{11e}$$

$$k_{DD22} = k_2\,l^2 + k_2(L-l)^2, \tag{11f}$$

where k_1 and k_2 parameterize the components of the bearing stiffness of the shaft in $\vec{n}_1$ and $\vec{n}_2$ direction respectively. Note that for $l = L/2$ or $a = 0$ the terms c_{TD11}, c_{TD22}, k_{TD11}, and k_{TD22} vanish meaning that translatory and rotational equations decouple as is intuitively clear and has already been noted at the end of Section 2. The resulting stiffness matrix will in this case decouple as

$$\mathbf{K}_{\text{dec.}} = \text{diag}\left(\left[\mathbf{C}_{TT}^{-1} + \mathbf{K}_{TT}^{-1}\right]^{-1}, \left[\mathbf{C}_{DD}^{-1} + \mathbf{K}_{DD}^{-1}\right]^{-1} \right) \tag{12}$$

For further analysis we have to specify the generalized coordinates. In order to obtain equations of motion of the simplest possible form we define the displacements q_1 and q_2 of the disc in non-rotating coordinates. For the rotational motion of the disk including tilting we define an Euler rotation by the angles φ, q_3, and q_4 about the body fixed axes $\vec{d}_3$, $\vec{d}_2$, and $\vec{d}_1$. For the rotation we introduce the constraint $\vec{\omega} = \Omega\vec{n}_3$ which in the linearized equations yields $\dot{\varphi} = \Omega$. The full state vector therefore reads $\mathbf{q}^T = [q_1, q_2, q_3, q_4]$, where q_1 and q_2 denote the translatory displacements, and q_3 and q_4 denote the tilting angles of the disk.

The equations of motion are finally derived using the symbolic manipulator Autolev [18] and read

$$\mathbf{M}\ddot{\mathbf{q}} + [(\mathbf{K}(t)^{-1} + \mathbf{C}(t)^{-1})^{-1} + \mathbf{K}_0 + \mathbf{N}]\mathbf{q} = \mathbf{0}, \tag{13}$$

where

$$\mathbf{M} = \text{diag}\left(m, m, \tfrac{1}{4}mr^2, \tfrac{1}{4}mr^2\right), \tag{14a}$$

$$\mathbf{K}_0 = \text{diag}\left(0, 0, \tfrac{1}{4}m\Omega^2 r^2, \tfrac{1}{4}m\Omega^2 r^2\right), \tag{14b}$$

$$\mathbf{N} = \text{diag}\left(\begin{bmatrix} 0 & n_T \\ -n_T & 0 \end{bmatrix}, \begin{bmatrix} 0 & n_D \\ -n_D & 0 \end{bmatrix} \right) \tag{14c}$$

and

$$\mathbf{K}=\begin{bmatrix} k_{TT11} & 0 & k_{TD11}\,c & -k_{TD11}\,s \\ 0 & k_{TT22} & k_{TD22}\,s & k_{TD22}\,c \\ k_{TD11}\,c & k_{TD22}\,s & k_{DD11}-\Delta k_{DD}\,s^2 & -\Delta k_{DD}\,s\,c \\ -k_{TD11}\,s & k_{TD22}\,c & -\Delta k_{DD}\,s\,c & k_{DD22}+\Delta k_{DD}\,s^2 \end{bmatrix}, \tag{14d}$$

$$\mathbf{C}=\begin{bmatrix} c_{TT11}-\Delta c_{TT}\,s^2 & \Delta c_{TT}\,s\,c & c_{TD11}\,c & -c_{TD22}\,s \\ \Delta c_{TT}\,s\,c & c_{TT22}+\Delta c_{TT}\,s^2 & c_{TD11}\,s & c_{TD22}\,c \\ c_{TD11}\,c & c_{TD11}\,s & c_{DD11} & 0 \\ -c_{TD22}\,s & c_{TD22}\,c & 0 & c_{DD22} \end{bmatrix}. \tag{14e}$$

In the expressions (14a) we use

$$\begin{aligned} \Delta k_{DD} &= k_{DD11}-k_{DD22}, \\ \Delta c_{TT} &= c_{TT11}-c_{TT22}, \\ s &= \sin(\Omega t), \\ c &= \cos(\Omega t). \end{aligned}$$

4. Perturbation problem

In order to obtain some analytic insight into the problem we assume that the deviations from a fully symmetric problem depend smoothly on a perturbation parameter ε, with the perturbations vanishing at $\varepsilon=0$. This means that we have

$$k_{TT,DD;11,22}=k_{TT,DD}\pm\Delta k(\varepsilon), \tag{15a}$$

$$c_{TT,DD;11,22}=c_{TT,DD}\pm\Delta c(\varepsilon), \tag{15b}$$

$$l=\frac{L}{2}+a(\varepsilon), \tag{15c}$$

$$n_T=n_T(\varepsilon), \tag{15d}$$

$$n_D=n_D(\varepsilon). \tag{15e}$$

Since the parameters are smooth, each of them can be expanded into a Taylor series $p(\varepsilon)=\varepsilon p_1+\cdots$. It follows that the equations of motion of our system can be written in the form

$$\mathbf{M}\ddot{\mathbf{q}}+\varepsilon\Delta\mathbf{D}(t)\dot{\mathbf{q}}+(\mathbf{K}_0+\varepsilon\Delta\mathbf{K}(t))\mathbf{q}=\mathbf{0}, \tag{16a}$$

$$\Delta\mathbf{K}(t)=\Delta\mathbf{K}\left(t+\frac{2\pi}{\Omega}\right) \tag{16b}$$

and

$$\mathbf{M}=\operatorname{diag}\left(m,m,\tfrac{1}{4}mr^2,\tfrac{1}{4}mr^2\right), \tag{16c}$$

$$\mathbf{K}_0=\operatorname{diag}\left(m\omega_T^2,m\omega_T^2,\tfrac{1}{4}mr^2\omega_D^2,\tfrac{1}{4}mr^2\omega_D^2\right). \tag{16d}$$

The matrix $\Delta\mathbf{D}(t)$ represents small, possibly time dependent terms proportional to the generalized velocities and is used to phenomenologically introduce a small damping as described later. Note that ω_T and ω_D are the eigenfrequencies of the unperturbed system. The equations of motion form a system of equations with periodic coefficients of which stability can be analyzed using Floquet theory. In this case we will take advantage of the fact that the unperturbed problem has constant coefficients and that in the unperturbed case translatory and rotational equations decouple.

In order to apply Floquet theory we write the equations in the form of a first-order system:

$$\dot{\mathbf{x}}=\mathbf{A}(t)\mathbf{x}, \tag{17}$$

with

$$\mathbf{x}=\begin{pmatrix}\mathbf{q}\\ \dot{\mathbf{q}}\end{pmatrix},\quad \mathbf{A}(t)=\begin{pmatrix}\mathbf{0} & \mathbf{I}\\ -\mathbf{M}^{-1}\mathbf{K}(t) & \mathbf{0}\end{pmatrix}, \tag{18}$$

where $\mathbf{0}$ is the 4×4 zero matrix. When asymmetry is small, in the first-order approximation we have

$$\mathbf{A}(t)=\mathbf{A}_0+\mathbf{A}_1(t), \tag{19}$$

$$\mathbf{A}_0 = \begin{pmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K}_0 & \mathbf{0} \end{pmatrix}, \quad \mathbf{A}_1(t) = \begin{pmatrix} \mathbf{0} & \mathbf{0} \\ -\mathbf{M}^{-1}\Delta\mathbf{K}(t) & \mathbf{0} \end{pmatrix}, \tag{20}$$

where $\mathbf{A}_0$ describes the unperturbed system and $\mathbf{A}_1$ is a small perturbation.

For stability analysis of the time-periodic system (17), one has to compute the Floquet matrix $\mathbf{F} = \mathbf{X}(T)$, where $\mathbf{X}(t)$ is the 8×8 matrix solving the system

$$\dot{\mathbf{X}} = \mathbf{A}(t)\mathbf{X}, \quad \mathbf{X}(0) = \mathbf{I} \tag{21}$$

with the 8×8 identity matrix as the initial condition. The system is asymptotically stable if $|\rho| < 1$ for all eigenvalues (also called Floquet multipliers) ρ of the matrix $\mathbf{F}$, see [14, Section 9.2]. If there is a multiplier with $|\rho| > 1$, the system is unstable. When $|\rho| \leq 1$ for all multipliers, then the system is stable if and only if all the multipliers with $|\rho| = 1$ are simple or semi-simple. Recall that semi-simple means that the eigenvalue is multiple but possesses as much linearly independent eigenvectors as its algebraic multiplicity.

The Floquet matrix is a smooth function of the parameters. We can write the perturbed Floquet matrix keeping only the first-order (linear) correction term as

$$\mathbf{F} = \mathbf{F}_0 + \mathbf{F}_1, \tag{22}$$

with [14, Section 9.3]

$$\mathbf{F}_1 = \mathbf{F}_0 \int_0^T \mathbf{X}_0^{-1}\mathbf{A}_1\mathbf{X}_0 \, \mathrm{d}t, \tag{23}$$

where T is the rotation period, $\mathbf{X}_0(t)$ is the matricant of the unperturbed system with $\mathbf{F}_0 = \mathbf{X}_0(T)$.

Since the unperturbed problem features constant coefficients and a symmetric mass and stiffness matrix with multiple eigenvalues, the multipliers ρ of the matrix $\mathbf{F}_0$ are semi-simple. A small perturbation of a multiplier ρ is denoted as

$$\rho = \rho_0 + \rho_1 + o(\rho_1), \quad \rho_1 = \rho_0 \eta, \tag{24}$$

where ρ_1 is the leading correction term and η is introduced for further convenience.

For absolute values of the perturbed multipliers (24), where $|\rho_0| = 1$, we have

$$|\rho| = |1 + \eta + o(\eta)| = 1 + \mathrm{Re}(\eta) + o(\eta). \tag{25}$$

For asymptotic stability, we need $|\rho| < 1$ and, hence,

$$\mathrm{Re}\,\eta < 0. \tag{26}$$

The imaginary part of η only affects higher-order terms in (25) and, thus, it does not contribute to the first-order stability condition (26).

The splitting of the Floquet multipliers depends on the multiplicity of the Floquet multipliers in the unperturbed problem. For our problem we use the fact that the equations of motion decouple into 4×4 systems which have exactly the same mathematical structure as the system analyzed in [17]. In the unperturbed case the translatory system is the same as the one analyzed in [17]. For the equations for the tilting of the disk, the equations have the same structure, the only difference being that the influence of static and rotating asymmetries in the stiffness reverses. This can be seen from (12) and with the parameters from (10) and (11). From this it follows that the eigenvectors of the translatory/tilting system $\mathbf{u}_{1;T,D}$, $\mathbf{u}_{2;T,D}$, $\overline{\mathbf{u}}_{1;T,D}$, and $\overline{\mathbf{u}}_{2;T,D}$ are exactly the same as the ones calculated in [17] augmented by zeros in the positions corresponding to the tilting/translatory system. Of course the same holds true for the eigenvectors $\mathbf{v}_{i;T,D}$ of the adjoint system.

In the non-resonant case $\Omega \neq \omega_{T,D}/n$, there are two values of ρ_1 describing splitting of the double multiplier ρ. These two values are determined as the eigenvalues of the 2×2 matrix [14, Section 9.7]:

$$\det\begin{pmatrix} \mathbf{v}_{1;T,D}^{\mathrm{T}}\mathbf{F}_1\mathbf{u}_{1;T,D} - \rho_1 & \mathbf{v}_{1;T,D}^{\mathrm{T}}\mathbf{F}_1\mathbf{u}_{2;T,D} \\ \mathbf{v}_{2;T,D}^{\mathrm{T}}\mathbf{F}_1\mathbf{u}_{1;T,D} & \mathbf{v}_{2;T,D}^{\mathrm{T}}\mathbf{F}_1\mathbf{u}_{2;T,D} - \rho_1 \end{pmatrix} = 0. \tag{27}$$

For resonant speeds $\Omega = \omega_{T,D}/n$, $\omega_T \neq \omega_D$ the Floquet multipliers will be determined by a 4×4 system corresponding either to the translatory or the rotational equations as analyzed in [17]. Note that only in the very degenerate case $\Omega = \omega_T/n = \omega_D/n$ the first approximation term of the Floquet multiplier will depend on the coupling terms between the translatory and the tilting systems $\mathbf{K}_{TD}$ and $\mathbf{C}_{TD}$. In this case the first approximation of the Floquet multipliers would be determined by an 8×8 matrix involving all eigenvectors of the unperturbed system.

The analysis reveals that for a small displacement of the disk from the middle of the shaft the stability regions of the translatory and tilting dof can be determined by superposition. Due to reversed influence of rotating and stationary asymmetries the resulting stability region will be small and in practice not easy to find. This means that for rotating systems for which neither the dof orthogonal nor the ones parallel to the axis of rotation are dominant, the achievement of a design robust against self-excited vibrations remains a difficult task. This conclusion will be supported by numerical calculations provided in the next section.

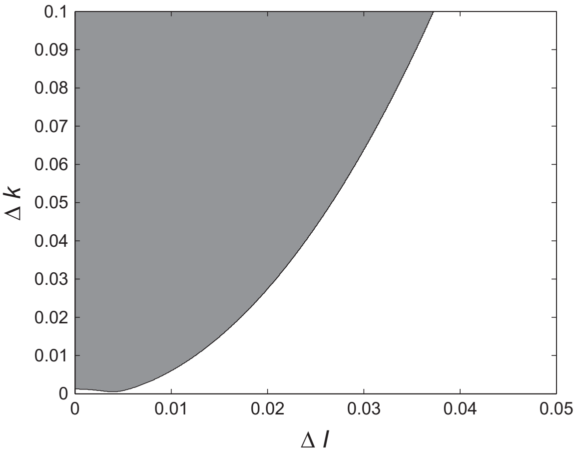

Fig. 2. Stability region (gray) of the translatory dof.

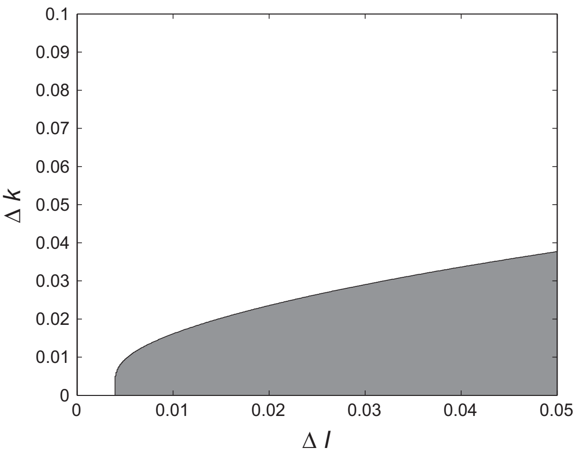

Fig. 3. Stability region (gray) of the tilting dof.

5. Numerical results

For the numerical studies we assume in accordance to the perturbation theory described in Section 4 that the basic moment of inertia of the rotating shaft is given by $I=(I_1+I_2)/2$ and its asymmetry is then determined by $\Delta I=(I_2-I_1)/2$. In direct analogy the stiffness of the bedding is determined by $k=(k_1+k_2)/2$ and the asymmetry parameter is $\Delta k=(k_2-k_1)/2$. In the following numerical analysis, we study the effect of the asymmetry parameters ΔI and Δk on the stability of the rotor. For numerical calculations we use the parameters

$$\begin{gathered}\Omega = 2\pi\ 1.5\ \text{rad/s},\\ L = 1\ \text{m},\\ m = 20\ \text{kg},\\ r = 0.5\ \text{m},\\ E = 210\ \text{GPa},\\ k = 10^5\ \text{N/m},\\ I = 8\times 10^{-9}\ \text{m}^4.\end{gathered}$$

The entries of the circulatory matrix have been chosen as $n_T=20$ N/m and $n_D=40$ N/m such that they are small compared to the restoring terms. In order to take into account small dissipation, a damping matrix $\mathbf{D}$ has been chosen as stiffness proportional as $\mathbf{D}=\alpha\mathbf{K}_0$ with $\alpha=10^{-7}$ 1/s.

The numerical integration of system (21) for the Floquet matrix $\mathbf{F}$ has been conducted in Matlab using the ode113 integrator with high accuracy. Instability of the system is determined when there is a multiplier $|\rho|>1+10^{-8}$, with the numerical tolerance 10^{-8} chosen for a robust estimation.

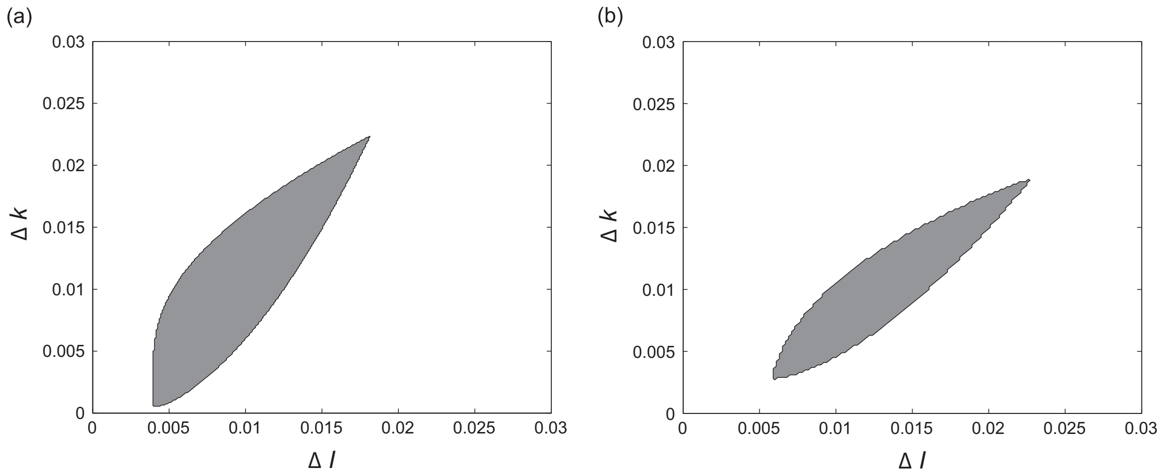

Fig. 4. (a) Superposition of stability regions for translatory and tilting dof ($a=0$). (b) Stability region for an unbalanced rotor with $a=0.005$.

First, the system is analyzed for a parameter $a=0$, when the system decouples into a translatory and a rotational part. If only the translatory dof are considered leading to a 4×4 matrix $\mathbf{A}(t)$, the resulting stability map is shown in Fig. 2.

It qualitatively gives the same results as for the system analyzed in [17]: While a rotating asymmetry ΔI destabilizes the system, the stationary asymmetry Δk has the contrary, namely stabilizing effect. We now consider only the rotational motion of the disk (also leading to a 4×4 $\mathbf{A}(t)$ matrix), where the effects of rotating and stationary asymmetries invert, as can be seen in the stability map in Fig. 3. The rotating asymmetry stabilizes the system, while it is destabilized by a stationary asymmetry in the bearings.

If we now analyze the model with all 4 dof, from the results of the previous section we expect that the stability regions of the rotor are determined by a superposition of the stability regions of the translatory and rotating dof for small $a(\varepsilon)$. The stability map of the 4 dof system is indeed given by a superposition of the stability maps shown in Fig. 2 for the translatory parts and in Fig. 3 for the rotating parts and is presented in Fig. 4a. When we choose a parameter $a(\varepsilon)=0.005$ to represent a disk with a slight deviation from the middle, we obtain the stability map presented in Fig. 4b. We see that the stable region becomes smaller with increasing a until it completely vanishes. The rotor is then completely unstable and neither a rotating nor a non-rotating asymmetry is helpful to avoid self-excited vibrations. This fact makes it difficult to select asymmetries to stabilize the system for a given band of rotation speeds Ω.

6. Conclusion

In this paper we considered the combined effect of rotating and non-rotating asymmetries on a rotor under non-conservative loading. It was shown that breaking of symmetries of the rotating and non-rotating restoring terms has different effects for translatory and rotational dof. For translatory dof, i.e., dof acting orthogonal to the axis of rotation, a non-rotating asymmetry has a stabilizing effect whereas a rotating asymmetry has a destabilizing effect. For the rotational dof of the rotor, i.e., dof acting parallel to the axis of rotation, the influence of stationary and rotating asymmetry is inverted. For weak coupling between translatory and rotational dof, depending on the parameters, a small stability region can still be found. For strong coupling, however, the system is always unstable as can be seen from the numerical calculations.

The results show that for general rotating systems it will be very difficult to find robust designs against self-excited vibrations using the approach of breaking symmetries since in- and out-of-plane vibrations couple due to the contact forces. From the structure of the equations studied it is quite obvious that the observed effects will also carry over to more complex rotor systems including systems governed by continuous models. Fortunately in many technical systems either the modes orthogonal or the modes parallel to the axis of rotation are dominant. Therefore in many cases breaking symmetries will still be a robust option for design engineers to avoid self-excited vibrations.

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